Problem Set 5 Answers January 2014

[1]

(1) $b > 0$ に対して $D_b : b^2 \le x^2 + y^2 \le 1$ とおく。

$I = \lim_{b \to 0+0} \iint_{D_b} \frac{1}{\sqrt{x^2+y^2}^a} dxdy.$ 極座標に直して計算して：$a \ge 2$ なら発散。$a < 2$ なら $I = \frac{2\pi}{2-a}$

(2) $b < 1$ に対して $D_b : x^2 + y^2 \le b^2$ とおくと

$I = \lim_{b \to 1-0} \iint_{D_b} \frac{1}{(x^2+y^2+1)^\alpha} dxdy.$ 極座標に直して計算して：$\alpha \le 1$ なら発散。$\alpha > 1$ なら $I = \frac{\pi}{4(\alpha-1)}$

[2] 別途

[3]

(1) $D : -1 \le y \le 1, y^2 \le x \le 2 - y^2$。$z = -\frac{1}{2}x - y + \frac{5}{2} \Rightarrow z_x = -\frac{1}{2}, z_y = -1 \Rightarrow \sqrt{1 + z_x^2 + z_y^2} = \frac{3}{2}$。

よって

$S = \int\int_D \frac{3}{2} dxdy = \int_{-1}^{1} \int_{y^2}^{2-y^2} \frac{3}{2} dxdy = 4$

(2) $D : 0 \le x \le 2, 0 \le y \le 3x$。$z = -y + \frac{x^2}{2} \Rightarrow z_x = x, z_y = -1 \Rightarrow \sqrt{1 + z_x^2 + z_y^2} = \sqrt{2 + x^2}$。よって

$S = \int\int_D \sqrt{2+x^2} dxdy = \int_0^2 \int_0^{3x} \sqrt{2+x^2} dydx = 6\sqrt{6} - 2\sqrt{2}$

(3) $D : -\frac{1}{2} \le x \le \frac{1}{2}, -\frac{1}{2} \le y \le \frac{1}{2}$。曲面は上下２枚。よって片方の面積を計算して２倍すればよい。

$z = \sqrt{1 - x^2} \Rightarrow z_x = \frac{-x}{\sqrt{1-x^2}}, z_y = 0 \Rightarrow \sqrt{1 + z_x^2 + z_y^2} = \frac{1}{\sqrt{1-x^2}}$。

$S = 2 \int\int_D \frac{1}{\sqrt{1-x^2}} dxdy = 8 \int_0^{1/2} \int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dxdy = 4[\sin^{-1} x]_0^{1/2} = \frac{2\pi}{3}$

(4) 考えている曲面を xz 平面に射影したものを D とすると $D : x^2 + z^2 \le 1$。

$y = 1 - x^2 - z^2 \Rightarrow y_x = -2x, y_z = -2z \Longrightarrow$

$S = \int\int_D \sqrt{1 + 4x^2 + 4z^2} dxdz$。 xz 平面の極座標に直して計算すれば

$S = \int_0^{2\pi} \int_0^1 \sqrt{1 + 4r^2} r dr d\theta = \frac{\pi}{6}(5\sqrt{5} - 1)$

[4]

(1) 1, (2) $(\frac{\pi^3}{2})(1 - \cos 1)$, (3) 18, (4) $\frac{7}{6}$, (5) 0

[5] 領域の図は掲示参照。

$(a) = \int_{-1}^{1} \int_0^{1-x^2} \int_{x^2}^{1-z} dydzdx \quad (b) = \int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_{x^2}^{1-z} dydxdz \quad (c) = \int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} dxdydz$

[6] (1) $2/3$ (2) $20/3$, (3) 1

[7]

(1) $\dfrac{4\pi(\sqrt{2}-1)}{3}$, (2) $\pi(6\sqrt{2}-8)$, (3) $5\pi/12$ (4) 5π

[8]

(1) $3\pi/10$, (2) $\pi/3$, (3) 2π

[9]

(1) $\displaystyle\int_0^{\pi}\int_0^{2\sin\theta}\int_0^{4-r\sin\theta} dzrdrd\theta$,

(2) $\displaystyle\int_0^{1}\int_0^{x}\int_0^{2-y} dzdydx =$ 円柱座標に直して $\displaystyle\int_0^{\pi/4}\int_0^{\sec\theta}\int_0^{2-r\sin\theta} rdzdrd\theta$

[10]

(1) $8\pi/3$ (2) $9/4$ (3) $\pi/2$ (4) 16π

[11]

(1) $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos\phi}^{2}\rho^2\sin\phi d\rho d\phi d\theta = \frac{31\pi}{6}$, (2) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{1-\cos\phi}\rho^2\sin\phi d\rho d\phi d\theta = \frac{8\pi}{3}$,

(3) $\displaystyle\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2\cos\phi}\rho^2\sin\phi d\rho d\phi d\theta = \frac{\pi}{3}$, (4) $\displaystyle\int_0^{2\pi}\int_{\pi/3}^{2\pi/3}\int_0^{a}\rho^2\sin\phi d\rho d\phi d\theta = \frac{2\pi a^3}{3}$

[12]

$$\int\int\int_D (x^2y+3xyz)dxdydz = \int_0^3\int_0^2\int_1^2\Big(\frac{v}{3}+\frac{vw}{3u}\Big)dudvdw = 2+\log 8$$